

# Cross-Layer Design for Scheduling and Antenna Sharing in MIMO Networks

Ghassane Aniba and Sonia Aïssa

INRS-EMT

University of Quebec

Montreal (QC), CANADA

Email: {ghassane, aïssa}@emt.inrs.ca

**Abstract**—This paper formulates the scheduling problem in MIMO networks as a Generalized Assignment Problem (GAP), and advances a new cross-layer design for the scheduling of users and the assignment of their corresponding data to the available transmit antennas. The proposed scheduling and antenna sharing method, referred to as Fast Transmit Antenna Selection (FTAS), uses Adaptive Proportional Fairness (APF) mapping as a means to determine the user-antenna assignment that maximizes the network performance both in terms of throughput and fairness. The proposed scheduler is applied in a High Speed Downlink Packet Access (HSDPA) network, taking advantage of an inherent HSDPA characteristic, namely, the use of adaptive modulation and coding, while coping with the imposed maximum number of simultaneously supported codes and the absence of fast power control. Numerical results show that our scheduler provides up to 70% increase in total throughput compared to other scheduling schemes applied to HSDPA.

## I. INTRODUCTION

Multiple antennas at both ends of a transmission link establish a spatial MIMO system well known to considerably increase the performance of wireless networks through the extra dimension offered in the spatial domain. Two types of MIMO systems can be distinguished: spatial multiplexing (SM) systems which exploit the multiple antennas as a means to increase the information data rate, and spatial diversity (SD) systems that aim to increase the reliability of the information transmission [1].

It has previously been shown that the solution to throughput maximization in a single transmit antenna system is to transmit to one user at a time [2]. In this case, the function of the scheduler is to select at each transmission time interval (TTI) the user for which transmission would maximize a given objective function, whether it be solely throughput or a compound function of throughput and fairness. Under multiple antennas at the transmitter and SM, the scheduler needs not only to select the set of users to transmit to, an issue pertaining to the MAC layer, but also the antenna(s) over which the data associated to each user would have to be transmitted, which is basically a PHY issue. The scheduling problem becomes hence a user selection and an antenna assignment problem with the simplest solution consisting of scheduling transmissions in a round-robin fashion as considered in [3]. In [4], a more elaborate approach was proposed which consists in using the Hungarian method [5] to assign users data to the available transmit antennas. However, the suggested approach

is suboptimal and yields an optimal assignment only when the number of antennas is equal to the number of active users. In tackling the problem in a global and comprehensive manner, this paper formulates the scheduling problem in a MIMO multiuser cellular network, proposes a framework for MIMO scheduling that exploits multiuser diversity to jointly maximize throughput and fairness, applies the proposed framework to High Speed Downlink Packet Access (HSDPA) [6], and provides performance analysis and comparisons that take into account the main characteristics and constraints of the aforementioned system. For example, HSDPA user equipments (UE) are classified upon their capabilities in terms of the maximum number of simultaneous spreading codes and the modulation order a UE can support. Twelve categories are distinguished, eight of which (1–6, 11 and 12) are characterized by a processing capability of at most 5 simultaneous spreading codes. In practice, more than 70% of the UEs will belong to these categories.

In this work, the scheduling problem is addressed as a Generalized Assignment Problem (GAP) [7] with a formulation that aims the maximization of a network utility function defined as a function of individual utilities of each user. In practice, a scheduling strategy needs to optimize both throughput and fairness which are two conflicting objectives, given that forcing fairness decreases the system throughput and can result in a significant loss as the load of the system gets higher. Herein, we propose a new user utility function and advance a novel cross-layer scheduler design as a general solution for the GAP. The proposed scheduling algorithm, called Fast Transmit Antenna Selection (FTAS), is an augmenting path technique that solves the GAP under study and provides the optimal users/antennas mapping –specifying the set of selected users and the mapping between these users and the transmit antennas– that maximizes the total utility of the network. The FTAS decisions are based on the utility function of each user provided by the Adaptive Proportional Fairness mapping (APF) [8], which maps the channel quality of each link (transmit antenna-user) into a utility function defined so as to control the throughput-fairness tradeoff in an adaptive and efficient way. Indeed, through performance evaluation and analysis for different operating conditions, the FTAS is shown to maximize the network utility and yield high

network utilization in addition to its flexibility in adapting to the network characteristics and parameters, such as the antenna configuration (MIMO, SIMO), user propagating conditions, and number of active users. To emphasize this, we apply our proposed framework to packet transmission in HSDPA.

The remainder of this paper is structured as follows. In Section II, we formulate the scheduling problem as a GAP and present the principle of the APF mapping. Section III, provides a detailed description of the FTAS algorithm. Section IV presents simulation results when applying FTAS for transmission in MIMO HSDPA. Finally, concluding remarks are drawn in Section V.

## II. SCHEDULING AS A GENERALIZED ASSIGNMENT PROBLEM

In a time-shared MIMO system that exploits multiuser diversity, the function of the scheduler is to select, in each TTI, the set of users to transmit to, and determine the appropriate antenna over which the data of each user should be transmitted, given the objective of maximizing the system's performance both in terms of throughput and fairness. We formulate this problem as a Generalized Assignment Problem [7], a constrained optimization problem, whose solution is a Transmit Assignment Matrix that defines both the set of users to transmit to and the transmit antenna(s) selected for each of these users.

Consider a MIMO system with  $N_T$  transmit antennas and  $K_A$  active users<sup>1</sup> having data queued for transmission at the base station. Each UE is equipped with  $N_R$  receive antennas and implements maximum ratio combining (MRC). We assume the channel state information (CSI) between each transmit antenna  $i$  and UE  $j$  to be available at the transmitter through a zero-delay error-free feedback channel, and define the corresponding link by a user utility function  $u$ . Generalizing over all transmit assignment possibilities, we define the utility matrix  $U = [u_{i,j}]_{i,j=1}^{N_T, K_A}$ , where element  $u_{i,j}$  specifies the utility of choosing transmit antenna  $i$  for user  $j$ , that is, the utility that would be achieved if the data of the  $j^{\text{th}}$  user were to be transmitted on the  $i^{\text{th}}$  antenna.

The generalized assignment problem can then be formulated as the maximization of the total utility of the system according to:

$$\max \sum_{i=1}^{N_T} \sum_{j=1}^{K_A} \lambda_{i,j} u_{i,j} \quad (1)$$

$$\sum_{i=1}^{N_T} \lambda_{i,j} \leq 1, \quad \forall j \in \{1, \dots, K_A\}; \quad (2)$$

$$\sum_{j=1}^{K_A} \lambda_{i,j} \leq K_{\max}, \quad \forall i \in \{1, \dots, N_T\}; \quad (3)$$

$$\lambda_{i,j} \in \{0, 1\}, \quad (4)$$

where constraint (2) translates the hypothesis that a user can be assigned to only one transmit antenna, constraint (3)

expresses the condition that a maximum of  $K_{\max}$  users can be assigned to each transmit antenna, and  $\lambda_{i,j}$  is the variable indicating whether user  $j$  is assigned to the  $i^{\text{th}}$  antenna or not. Finally, denote by  $K_S$  the maximum number of users that can be simultaneously be served by the base station ( $K_S = K_{\max} \times N_T$ ).

Constraint (3) can be seen as a simplification to the problem. Indeed, in the general case the limit on the number of antennas that could be assigned to a user would be  $N_T$ , a configuration that requires a huge amount of uplink signaling in order to make an optimal assignment. Nevertheless, the constraint considered herein doesn't induce a significant decrease in the total utility of the network, as observed in prior studies [9]–[11] where the selection of one transmit antenna for each user combined with MRC at the receiver side, was shown to achieve a full channel diversity ( $N_R \times N_T$ ), the same diversity order as using all the  $N_T$  transmit antennas.

The solution to the problem is defined as the Transmit Assignment Matrix (TAM)

$$G = [\lambda_{i,j}]_{i,j=1}^{N_T, K_A}, \quad (5)$$

that maximizes the objective function (1) subject to the identified constraints (2)-(4). Note that for convenience, the TTI index is dropped from the formulation.

Given the objective of maximizing throughput while ensuring fairness among users, the function of the scheduler is to find the best TAM over short time scales (TTI), and to determine how resources should be shared among users over longer time scales. To appropriately define long-term sharing objectives, it is necessary to define a utility function that not only ensures a reasonable compromise between throughput and fairness but also accounts for the time-variability in terms of transmission demands and propagating conditions. To this end, we define a utility function that ensures Adaptive Proportional Fairness (APF) among the active users [8]. The APF utility function corresponding to the pair ( $i^{\text{th}}$  transmit antenna,  $j^{\text{th}}$  UE) is given by

$$u_{i,j} = \frac{(r_{i,j})^{e_j}}{R_j}, \quad (6)$$

where  $r_{i,j}$  is the instantaneous rate between antenna  $i$  and user  $j$ ,  $R_j$  is the throughput of user  $j$ , and  $e_j$  is a weighted exponent used to control the proportional allocation between users so as to provide adaptive proportional fairness among them. Updating of the parameters  $R_j$  and  $e_j$  is detailed in [8]. Furthermore, we define the maximum throughput that could be reached by a user  $j$  as

$$\bar{r}_j = \frac{1}{N_T} \sum_{i=1}^{N_T} \bar{r}_{i,j}, \quad (7)$$

where  $\bar{r}_{i,j}$  represents the average data rate that could be achieved over the channel between antenna  $i$  and user  $j$  if the latter were to be selected at each TTI. This value can be updated at each TTI by averaging  $r_{i,j}$ , or represented by the data rate that is equivalent to the average  $\overline{SNR}_{i,j}$  over the

<sup>1</sup>The terms "user" and "user equipment (UE)" are used interchangeably.

channel (transmit antenna  $i$ , user  $j$ ). Hence, the APF module generates the utility matrix  $U = [u_{i,j}]_{i,j=1}^{N_T, K_A}$  that will be used by the Fast Transmit Antenna Selection (FTAS) algorithm to determine the optimal TAM,  $G^*$ .

### III. FAST TRANSMIT ANTENNA SELECTION

The optimization problem under consideration can be formulated as finding, in each TTI, the TAM that maximizes the total utility of the system (1) under the identified constraints (2)-(4). In this section, we present our solution to the problem, namely, the Fast Transmit Antenna Selection algorithm which is characterized by its flexibility in adapting to practical transmission scenarios and configurations in terms of number of transmit antennas  $N_T$ , number of active users  $K_A$ , and/or the maximum number of users that can be assigned to a transmit antenna, which is particularly important when UEs have different resource requirements and capabilities as in HSDPA.

FTAS is an *Augmenting Path* technique that determines the optimal TAM  $G^*$  for a given TTI, taking as input information the utility matrix  $U$  produced by the APF mapping. To each element  $u_{i,j}$  of  $U$ , the FTAS finds the corresponding value in  $G^*$ , namely  $\lambda_{i,j} \in \{0, 1\}$  with the value  $\lambda_{i,j} = 1$  indicating that UE  $j$  is selected for the current TTI and that its data is to be transmitted on antenna  $i$ . Under the constraint that a user can be assigned to only one transmit antenna (2), each column of  $G^*$  can have only one non-zero value to which corresponds an utility value that contributes to the total utility achieved in the current TTI. Consequently, the FTAS can be seen as a search in  $U$  for the path specifying the positions of elements  $u_{i,j}$  that maximize the objective function, that is, the sequence of positions with non-zero  $\lambda_{i,j}$ .

In the following, we describe the algorithm by distinguishing two operating conditions that are represented by the relationship between the *offer* and the *demand*. The offer is expressed as the maximum number of users that can simultaneously be served,  $K_S$ , and the demand is represented by the number of users awaiting transmission,  $K_A$ . In the first case, the offer is considered higher than the demand, i.e.,  $K_S \geq K_A$ , and in the second, the demand is higher than the offer, i.e.,  $K_A > K_S$ . The parameter  $K_S$  being directly related to the number of transmit antennas  $N_T$  and the matrix  $U$  having size  $N_T \times K_A$ , the dimension that would be fully used by the algorithm is  $K_A$  in the first case, and  $N_T$  in the second. Denote by  $D$  the data matrix with number of columns the value that would be fully used by the FTAS algorithm, hence  $D$  is equal to  $U$  or its transpose  $U^T$ , and consider the following notation and terminology:

- $m$ : index of iteration;
- $M$ : number of iterations;
- $D^{(m)} = [d_{l,c}]_{l,c=1}^{A^{(m)}, B}$ : the data matrix in iteration  $m$ , with number of columns the value that would fully be used by the algorithm;
- $P_n(k) = \{(l_1^n, 1), (l_2^n, 2), \dots, (l_k^n, k)\}$ : path  $n$  from column 1 to column  $k$  in matrix  $D^{(m)}$ , where each element

$(l_c^n, c)$  consists of the pair (line index, corresponding column);

- $L_n(k) = \{l_1^n, l_2^n, \dots, l_k^n\}$ : sequence of line indices corresponding to the  $n^{\text{th}}$  path  $P_n(k)$ , with  $C(x)$  the number of times a value  $x$  appears in the sequence  $L_n(k)$ ;
- $S(k) = \cup P_n(k)$ : list of surviving paths at column  $k$ , with cardinality  $\mathcal{C}(S(k))$ ;
- $Z^{(m)}$  group of surviving paths  $S(B)$  corresponding to iteration  $m$ , with cardinality  $\mathcal{C}(Z^{(m)})$ ;
- $T(P_n(k)) = \sum_{(l,c) \in P_n(k)} d_{l,c}$ : weight of the  $n^{\text{th}}$  path  $P_n(k)$ .

Starting with a set of initial conditions, the algorithm iterates on  $m$  until the stopping condition is met and the optimal TAM found. The FTAS is presented in its general form in Algorithm-1. Besides that, Fig. 1 presents a diagram that describes in detail the operations of Algorithm-1 considering the two aforementioned cases. Indeed, depending on the relationship between  $K_S$  and  $K_A$ , FTAS executes a number of compulsory steps,  $M = 1$  times if  $K_S \geq K_A$ , and  $M = K_{\max}$  times when  $K_A > K_S$ . Moreover, FTAS includes some optional steps (11, 13 and 26) which are introduced to determine  $G^*$  while avoiding unnecessary computations, thus limiting the complexity of the approach. In particular, the first optional step represents permutation of the data matrix columns so as to position them in the descending order in terms of the highest utility value of each column  $k$ , ( $\max_{l=1, \dots, A^{(m)}} d_{l,k}$ ), and the last optional steps consist in using the metric

$$T_r^{(m)}(k) = \sum_{s=k+1}^B \max_{l=1, \dots, A^{(m)}} d_{l,s}, \quad (8)$$

defined as the maximum remaining weight, to quickly eliminate paths that would not lead to the optimal solution, i.e., the optimal TAM.

In each iteration  $m$ , the data matrix  $D^{(m)}$  and the metric  $T_r^{(m)}(k)$  are updated based on the result of the previous iteration. Specifically,  $D^{(m)}$  is a sub-matrix of  $D^{(m-1)}$  generated by deleting in the latter, the lines already used in the optimal path found at iteration  $m-1$ , with the exception that  $D^{(0)} = U$  in first case ( $K_S \geq K_A$ ) and  $D^{(0)} = U^T$  in the second ( $K_A > K_S$ ).

We represent in Fig. 2, an example when the number of active users ( $K_A = 6$ ) is less than the number of possible simultaneous transmissions ( $K_S = 8$ ). Fig. 2.a shows the initial input matrix  $U$  on which the first optional step (permutation of columns) is performed (Fig. 2.b), followed by the  $T_r$  computation for each column. As can be seen, execution of the FTAS algorithm produces two paths (Bold line and Dotted line) with the same total utility value. Hence, two transmit assignment matrices

$$G_1^* = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad G_2^* = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

can be chosen in this case. Here it is important to mention that even though multiple maxima may exist, non-uniqueness of the TAM does not represent a significant practical concern given that the procedure is executed at each TTI and that a user that has not been selected through a choice for the TAM would likely be in the next TTI.

#### IV. PERFORMANCE EVALUATION

Hereafter, we present simulation results for the proposed scheduler and evaluate its performance in comparison with two scheduling approaches, namely, the Proportional Fairness (PF) policy [12], and the best-user method [13]. The performance analysis is performed for packet transmission in HSDPA, in terms of fairness and throughput. Consider first the SISO configuration ( $N_T \times N_R = 1 \times 1$ ) and a simple simulation scenario, chosen to assess the performance of FTAS compared to the aforementioned policies, and show its high efficiency in improving fairness between users even when these exhibit heterogeneous propagating conditions. We consider eight active users (UE<sub>1</sub>–UE<sub>8</sub>) and assume that all are allocated the same transmission power, the only difference being the type of variations exhibited by the channel of each user (Rayleigh with  $f_d = 5$ Hz, Shadowing, or both). The distributions corresponding to the channels of the different users are chosen so as the variability gets higher as the UE index increases.

We present in Fig. 3 comparison of the normalized throughput ( $\frac{R_j}{\bar{r}_j}$ ,  $j=1, \dots, 8$ ) achieved by using the PF and the FTAS. For a fair comparison, we consider that two users can be selected based on the PF decision rule [12]. As can be observed, our algorithm outperforms the PF method through a fair throughput allocation despite the difference in the channel distributions of the different users or the underlying variances. For example, examining these results shows how the PF policy allocates to UE<sub>8</sub> 46% of his maximum average data rate,  $\bar{r}_j$ , whereas UE<sub>1</sub> gets allocated only 18% of the data rate his channel can support, a value that represents less than half the share corresponding to UE<sub>8</sub>. On the other hand, using FTAS, the allocation is between 26% and 28% for all users. Hence, compared to the PF policy, the FTAS algorithm achieves fairness between users with different channel's variability in terms of variance and that with a loss of only 10% in total throughput.

We consider now the MIMO configuration and evaluate the throughput performance of the proposed method as compared with the above-mentioned approaches. The algorithm has been studied for different antenna configurations. Herein, results are provided for antenna settings where the number of transmit antennas  $N_T \in \{1, 2, 4\}$  and the number of receive antennas  $N_{R_j} = 1$  or 2, for  $j = 1, \dots, K_A$  with  $K_A = 12$ , and UEs belonging to category 6 [14]. Results corresponding to the SISO configuration are also presented to serve as a reference point. As for the channel, we consider correlated Rayleigh fading based on the MIMO channel model presented in [15]. We also consider slowly-moving users (3km/h) uniformly distributed within the cell. Comparing the total throughput

( $\sum_{j=1}^{K_A} R_j$ ) obtained when using the FTAS algorithm (Fig. 4), with that obtained using the method proposed in [13] in which the best-user (having the best channel quality) is selected for each antenna (Fig. 5), shows that FTAS outperforms the best-user method. For example, when two antennas are employed at the base as well as at the UEs ( $2 \times 2$ ), the use of FTAS yields a throughput 6Mb/s higher than that obtained by the best-user method, and an increase of 3Mb/s for the SISO configuration. Indeed, in the SISO case, due to the fact that all users are limited by 5 simultaneous codes [14], the best-user method reaches the maximum throughput that could be achieved by a user ( $\frac{7168 \text{ bits}}{0.002 \text{ s}} = 3.5 \text{ Mb/s}$ ), while FTAS yields approximately the double (6.8Mb/s) by allowing transmission to two users over each antenna, hence maximizing user diversity and using all the available ten codes over each transmit antenna.

#### V. CONCLUSION

This paper formulated the scheduling problem in MIMO cellular networks as a Generalized Assignment Problem (GAP) and proposed a new MIMO scheduler as a general solution for the GAP using a cross-layer design approach. The proposed scheduler uses Adaptive Proportional Fairness (APF) mapping and implements a new Fast Transmit Antenna Selection (FTAS) technique to make an optimal selection and assignment of users to the available antennas. The proposed scheduler was applied for transmission in HSDPA and shown to enhance the system's throughput while providing an acceptable level of fairness between users, and that compared to both the Proportional Fairness (PF) method and the best-user scheduling.

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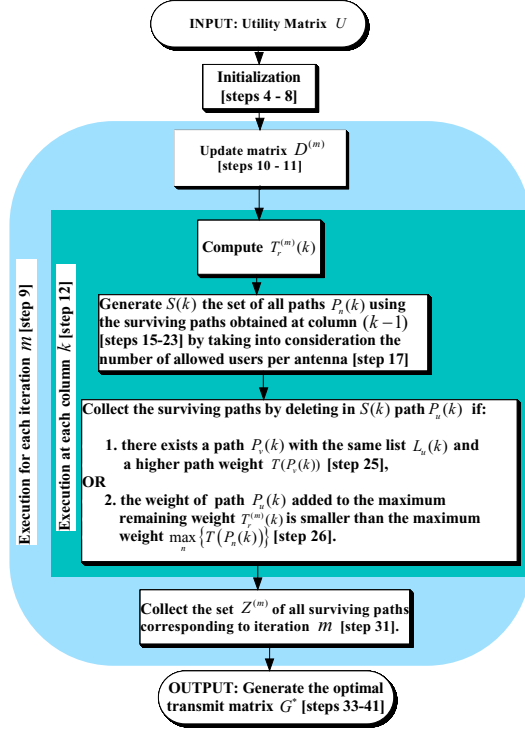


Fig. 1. FTAS diagram describing the main operations of Algorithm-1

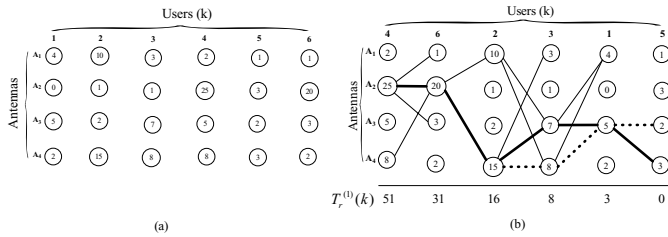


Fig. 2. An example illustrating the operation of FTAS when the number of active users  $K_A$  is less than the maximum number of simultaneous transmissions  $K_S$

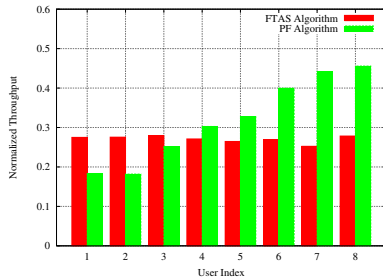


Fig. 3. Normalized throughput of each user: comparison between the PF and FTAS algorithms in the SISO configuration

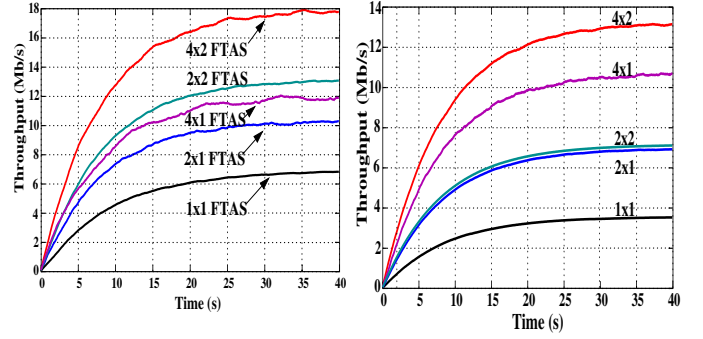


Fig. 4. The total throughput performance obtained using the Fast Transmit Antenna Selection (FTAS) scheduler. Fig. 5. The total throughput performance obtained using best-user scheduling.

### Algorithm 1 Fast Transmit Antenna Selection algorithm

- 1: **INPUT:** Utility Matrix  $U$ .
- 2: **OUTPUT:** Transmit Assignment Matrix  $G^*$ .
- 3: **INITIALIZATION:**
- 4:  $G^* \leftarrow [0]_{i,j}^{N_T, K_A}$ ,  $m = 0$
- 5: **if**  $K_S \geq K_A$  **then**  $D^{(0)} \leftarrow U$ ,  $M \leftarrow 1$
- 6: **else**  $D^{(0)} \leftarrow U^T$ ,  $M \leftarrow K_{\max}$
- 7:  $S(1) = \{(1, 1)\}, \{(2, 1)\}, \dots, \{(A^{(0)}, 1)\}$
- 8: **PROCEDURE:**
- 9: **for**  $m = 1$  **to**  $M$  **do**
- 10: Generate sub-matrix  $D^{(m)}$  by deleting in  $D^{(m-1)}$  lines already used in the optimal path of iteration  $(m - 1)$ ;
- 11: Order columns of  $D^{(m)}$  in the descending order in terms of the highest utility value in each column;
- 12: **for**  $k = 2$  **to**  $B$  **do**
- 13: Compute  $T_r^{(m)}(k)$
- 14:  $q \leftarrow 0$
- 15: **for**  $n = 1$  **to**  $\mathcal{C}(S(k-1))$  **do**
- 16: **for**  $l = 1$  **to**  $A^{(m)}$  **do**
- 17: **if**  $C(l) \leq M - 1$  **then**
- 18:  $P_q(k) \leftarrow \{P_n(k-1), (l, k)\}$
- 19:  $q \leftarrow q + 1$
- 20: **end if**
- 21: **end for**
- 22: **end for**
- 23:  $S(k) \leftarrow \bigcup_{n=1}^q P_n(k)$
- 24: **for**  $\{P_u(k), P_v(k)\} \in S(k)$  **do**
- 25: **if**  $[L_u(k) = L_v(k) \ \& \ T(P_u(k)) < T(P_v(k))]$   
 $\parallel [T(P_u(k)) < \max_n \{T(P_n(k))\} - T_r^{(m)}(k)]$
- 26: **then**
- 27:  $S(k) \leftarrow S(k) \setminus P_u(k)$
- 28: **end if**
- 29: **end for**
- 30: **end for**
- 31:  $Z^{(m)} \leftarrow S(B)$
- 32: **end for**
- 33: **for** each path  $P \in \bigcup_{p=1}^{C(Z^{(m)})} Z^{(p)}$  **do**
- 34: **for**  $(i, j) \in P$  **do**
- 35: **if**  $K_S \geq K_A$  **then**
- 36:  $\lambda_{i,j} \leftarrow 1$
- 37: **else**
- 38:  $\lambda_{j,i} \leftarrow 1$
- 39: **end if**
- 40: **end for**
- 41: **end for**